An Improved Speedup Factor for Sporadic Tasks with Constrained Deadlines under EDF

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Outline

• Problem statement
• Main result
• Sketchy proof
• Open problems
Periodic tasks and sporadic tasks

- A task: a sequence of jobs with execution time $e$ and deadline $d$

• Periodic tasks: fixed interarrival time $p$
Periodic tasks and sporadic tasks

- A task: a sequence of jobs with execution time $e$ and deadline $d$

- Periodic tasks: fixed interarrival time $p$
- Sporadic tasks: varying interarrival time $\geq p$

$\tau = (e, d, p)$
Schedulability testing of $\tau = \{\tau_1, \ldots, \tau_n\}$

- On a unit-speed uniprocessor
  - Resource demand:

![Diagram of resource demand](image)
Schedulability testing of $\tau = \{\tau_1, \ldots, \tau_n\}$

- On a unit-speed uniprocessor
  - Resource demand:
    $$dbf(\tau, t) = \sum_{\tau_i \in \tau} dbf(\tau_i, t)$$
  - Schedulable $\iff dbf(\tau, t) \leq t$
  - Co-NP-hard to check

Schedulability testing of $\tau = \{\tau_1, \ldots, \tau_n\}$

- On a unit-speed uniprocessor
  - Resource demand:
    \[ dbf(\tau, t) = \sum_{\tau_i \in \tau} dbf(\tau_i, t) \]
  - Approximation:
    \[ dbf^*(\tau_i, \cdot) dbf^*(\tau_j, \cdot) \]

Schedulability testing of $\tau = \{\tau_1, \ldots, \tau_n\}$

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  - Resource demand:
    $$dbf(\tau, t) = \sum_{\tau_i \in \tau} dbf(\tau_i, t)$$
  - Approximation:
    $$dbf^*(\tau, t) = \sum_{\tau_i \in \tau} dbf^*(\tau_i, t)$$

Schedulability testing of $\tau = \{\tau_1, \ldots, \tau_n\}$

- On a speed-1 multiprocessor
  - Partitioned scheduling (Any fit with dbf* works)
  - Speedup bound is $1 + \rho$ [a]

$$\rho = \sup_{\tau : dbf(\tau, t) \leq t} \frac{dbf^*(\tau, d)}{d}, d = \max\{d_1, \ldots, d_n\}$$

Schedulability testing of $\tau = \{\tau_1, \ldots, \tau_n\}$

- On a speed-1 multiprocessor
  - Partitioned scheduling (Any fit with dbf* works)
  - Speedup bound is $1 + \rho$ [a]
    \[
    \rho = \sup_{\tau: dbf(\tau,t) \leq t} \frac{dbf^*(\tau, d)}{d}, \quad d = \max\{d_1, \ldots, d_n\}
    \]

A schedulability test has **speedup factor** [b] $s$, $s \geq 1$, if any task set that is schedulable by any algorithm on platform with processors of speed 1, it will be deemed schedulable by this test upon a platform with processors that are $s$ times as fast.

- **Speedup bound** = lower bound of speedup factor
- Major metric & standard tool for evaluating sub-optimality

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Ultimate objective

- Figure out $\rho$
- Known results
  (Constrained-deadline)
Main result

• Better upper bound of $\rho$
  • Lower speedup factor of partitioned scheduling
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Main ideas to estimate $\rho$

- Principle: Keep $\tau$ feasible while not decreasing $\text{Dbf}^*(\tau,d_n)$

- Details: Fix many parameters
  - Identical execution times: $e_i = 1$
  - Tight deadlines: $d_i = d_{i-1} + e_i = i$
  - Confined periods: $n < p_i + d_i \leq 2n$

- Reduce the problem to an “easy” math programming

- Lossless
- Lossy
Step 0: Normalization

• Original objective

\[
\begin{align*}
\sup_{\text{subject to}} & \quad dbf^*(\tau, d_n) \frac{d_n}{d_n} , \\
& \quad dbf(\tau, t) \leq t, \quad \forall t > 0 \\
& \quad d_1 \leq d_2 \leq \cdots \leq d_n , \\
& \quad n \in \mathbb{Z}^+, e_i, d_i, p_i \in \mathbb{R}^+, \quad 1 \leq i \leq n.
\end{align*}
\]

• \( MP_0 \)

\[
\begin{align*}
\sup_{\text{subject to}} & \quad dbf^*(\tau, d_n) \frac{d_n}{d_n} , \\
& \quad dbf(\tau, t) \leq t, \quad \forall t > 0 \\
& \quad d_i + p_i > d_n , \quad 1 \leq i \leq n-1, \\
& \quad d_1 \leq d_2 \leq \cdots \leq d_n , \\
& \quad n \in \mathbb{Z}^+, e_i, d_i, p_i \in \mathbb{R}^+, \quad 1 \leq i \leq n.
\end{align*}
\]

Let $\tau = \{\tau_1, \ldots, \tau_n\}$ be a feasible solution to the original problem. Suppose $d_i + p_i \leq d_n$. Transform $\tau_i$ into $\tau'_i$

$$d_i \geq d_n$$

Transform $\tau_i$ into $\tau'_i$

$$\tau' = \{\tau'_1, \ldots, \tau'_n\}$$

is a feasible solution to $MP_1$

$$dbf(\tau, t) \geq dbf(\tau', t) \Rightarrow \tau' = \{\tau'_1, \ldots, \tau'_n\}$$

$$dbf^*(\tau, d_n) = dbf^*(\tau', d'_n) \Rightarrow \text{equal objective values}$$

Step 1: Rationalization

\[
\begin{align*}
\textbf{MP}_0 & \quad \sup \frac{\text{dbf}^*(\tau, d_n)}{d_n}, \\
\text{subject to} & \quad \text{dbf}(\tau, t) \leq t, \quad \forall \ t > 0 \\
& \quad d_i + p_i > d_n, \quad 1 \leq i \leq n - 1, \\
& \quad d_1 \leq d_2 \leq \cdots \leq d_n, \\
& \quad n \in \mathbb{Z}^+, e_i, d_i, p_i \in \mathbb{R}^+, \quad 1 \leq i \leq n.
\end{align*}
\]

\[
\begin{align*}
\textbf{MP}_1 & \quad \sup \frac{\text{dbf}^*(\tau, d_n)}{d_n}, \\
\text{subject to} & \quad \text{dbf}(\tau, t) \leq t, \quad \forall \ t > 0 \\
& \quad d_i + p_i > d_n, \quad 1 \leq i \leq n - 1, \\
& \quad d_1 \leq d_2 \leq \cdots \leq d_n, \\
& \quad n \in \mathbb{Z}^+, e_i, d_i, p_i \in \mathbb{Q}^+, \quad 1 \leq i \leq n.
\end{align*}
\]

Lossless due to continuity
Step 2: Tight deadlines

- $MP_1$

\[
\begin{align*}
\sup & \quad \frac{dbf^* (\tau, d_n)}{d_n}, \\
\text{subject to} & \quad dbf(\tau, t) \leq t, \quad \forall t > 0 \\
& \quad d_i + p_i > d_n, \quad 1 \leq i \leq n - 1, \\
& \quad d_1 \leq d_2 \leq \cdots \leq d_n, \\
& \quad n \in \mathbb{Z}^+, e_i, d_i, p_i \in \mathbb{Q}^+, \quad 1 \leq i \leq n.
\end{align*}
\]

- $MP_2$

\[
\begin{align*}
\sup & \quad \frac{dbf^* (\tau, d_n)}{d_n}, \\
\text{subject to} & \quad dbf(\tau, t) \leq t, \quad \forall t > 0 \\
& \quad d_i + p_i > d_n, \quad 1 \leq i \leq n - 1, \\
& \quad d_i = e_i + d_{i-1}, \quad 1 \leq i \leq n, \\
& \quad n \in \mathbb{Z}^+, e_i, d_i, p_i \in \mathbb{Q}^+, \quad 1 \leq i \leq n.
\end{align*}
\]
Step 3: Identical execution times

\[ MP_2 \]
\[
\begin{align*}
\mathop{\sup}_{d_n} \frac{\text{dbf}^*(\tau, d_n)}{d_n}, \\
\text{subject to} \quad \text{dbf}(\tau, t) \leq t, \quad \forall t > 0 \\
\quad d_i + p_i > d_n, \quad 1 \leq i \leq n - 1, \\
\quad d_i = e_i + d_{i-1}, \quad 1 \leq i \leq n, \\
\quad n \in \mathbb{Z}^+, e_i, d_i, p_i \in \mathbb{Q}^+, \quad 1 \leq i \leq n.
\end{align*}
\]

\[ MP_3 \]
\[
\begin{align*}
\mathop{\sup}_{d_n} \frac{\text{dbf}^*(\tau, d_n)}{d_n}, \\
\text{subject to} \quad \text{dbf}(\tau, t) \leq t, \quad \forall t > 0 \\
\quad d_i + p_i > d_n, \quad 1 \leq i \leq n - 1, \\
\quad d_i = e_i + d_{i-1}, \quad 1 \leq i \leq n, \\
\quad e_i = 1, \quad 1 \leq i \leq n \\
\quad n \in \mathbb{Z}^+, e_i, d_i, p_i \in \mathbb{Q}^+, \quad 1 \leq i \leq n.
\end{align*}
\]

- Proportionally scaling all the parameters keeps the feasibility and objective value.
- The original parameters are rational.
Decompose each task $\tau_i$ into task set $\{\tau_1^i, ..., \tau_k^i\}$
Decomposed task set is feasible to $MP_2$ by construction
\[ dbf^*(\tau_i, d_n) \leq \sum_{j=1}^{k} dbf^*(\tau_j^i, d_n) \]
\[ dbf^*(\tau_i, d_n) \leq \sum_{j=1}^{k} dbf^*(\tau_j^i, d_n) \]
Step 4: Confined range of periods

\[ M_{P3} \]

\[
\text{sup} \frac{dbf^*(\tau, d_n)}{d_n}, \quad \text{subject to}
\]
\[ dbf(\tau, t) \leq t, \quad \forall t > 0 \\
d_i + p_i > d_n, \quad 1 \leq i \leq n - 1, \\
d_i = e_i + d_{i-1}, \quad 1 \leq i \leq n, \\
e_i = 1, \quad 1 \leq i \leq n \\
n \in \mathbb{Z}^+, e_i, d_i, p_i \in \mathbb{Q}^+, \quad 1 \leq i \leq n.
\]

\[ M_{P4} \]

\[
\text{sup} \frac{dbf^*(\tau, d_n)}{d_n}, \quad \text{subject to}
\]
\[ n < d_i + p_i \leq 2n \text{ are distinct} \\
d_i = e_i + d_{i-1}, \quad 1 \leq i \leq n, \\
e_i = 1, \quad 1 \leq i \leq n \\
n \in \mathbb{Z}^+, e_i, d_i, p_i \in \mathbb{Q}^+, \quad 1 \leq i \leq n.
\]
Basic idea

• Let \( \tau = \{\tau_1, ..., \tau_n\} \) be a feasible solution to \( MP_3 \)

• For any task \( \tau_i = (e_i, d_i, p_i) \), suppose \( d_i + p_i \) is the \( j \)th smallest.
  • \( d_i + p_i \geq n + j \) since \( d_i + p_i \geq dbf(\tau, d_i + p_i) \geq n + j \)

• Transform \( \tau_i \) to \( \tau'_i = (e_i, d_i, p'_i) \) such that \( d_i + p'_i = n + j \)

• \( \tau' = \{\tau'_1, ..., \tau'_n\} \) is a feasible solution to \( MP_4 \)
\[ df^*(\tau_i, d_n) \leq df^*(\tau'_i, d_n) \]

- \( \rho \) is upper bounded by the optimum value \( \rho' \) of \( MP_4 \)
- \( \rho' \leq \sup_{p_1 + \cdots + p_n = n^2} \left( 2 - \frac{\sum_{i=1}^{n} i}{n} \right) \) \implies \( \rho' \leq \frac{14}{9} \)
- \( \sum_{i=1}^{n} x_i = n^2 \implies \sum_{i=1}^{n} \frac{i}{x_i} \geq \frac{4n}{9} \) \implies \( \rho \leq \frac{14}{9} \)

\[ \sup_{d_n} \frac{df^*(\tau, d_n)}{d_n}, \quad \text{subject to} \]
\[ n < d_i + p_i \leq 2n \text{ are distinct} \]
\[ d_i = e_i + d_{i-1}, \quad 1 \leq i \leq n, \]
\[ e_i = 1, \quad 1 \leq i \leq n \]
\[ n \in \mathbb{Z}^+, e_i, d_i, p_i \in \mathbb{Q}^+, \quad 1 \leq i \leq n. \]
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• Conclusion
Conclusion

• Demand/resource $\rho$

• Speedup factor of partitioned EDF: 2.5556.

• WiP: Arbitrary deadline (uniproc only)

$\rho$

2005, Baruah&Fisher

2011, Chen&Chakraborty

Our result
Thank You!

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A schedulability test has speedup factor \( s, s \geq 1 \), if any task set that is schedulable by any algorithm on platform with processors of speed 1, it will be deemed schedulable by this test upon a platform with processors that are \( s \) times as fast.

- Speedup bound means a lower bound of speedup factor
- Major metric & standard tool for evaluating sub-optimality
- Potential pitfalls


Let $\tau = \{\tau_1, ..., \tau_n\}$ be a feasible solution to $MP_1$. Chose the smallest $i$ s.t. $d_i \neq e_i + d_{i-1}$. Transform $\tau_i$ into $\tau_i'$

$$dbf(\tau_i; \cdot)$$

$$dbf(\tau_{i-1}; \cdot)$$

$$dbf(\tau_i'; \cdot)$$

$$dbf(\tau_{i-1}'; \cdot)$$

$dbf(\tau', t) \leq t$ holds by construction
\[ dbf(\tau_i, \cdot) \]

\[ dbf(\tau_i', \cdot) \leq dbf^*(\tau_i, d_n) \leq dbf^*(\tau_i', d_n) \]