# An Improved Speedup Factor for Sporadic Tasks with Constrained Deadlines under EDF


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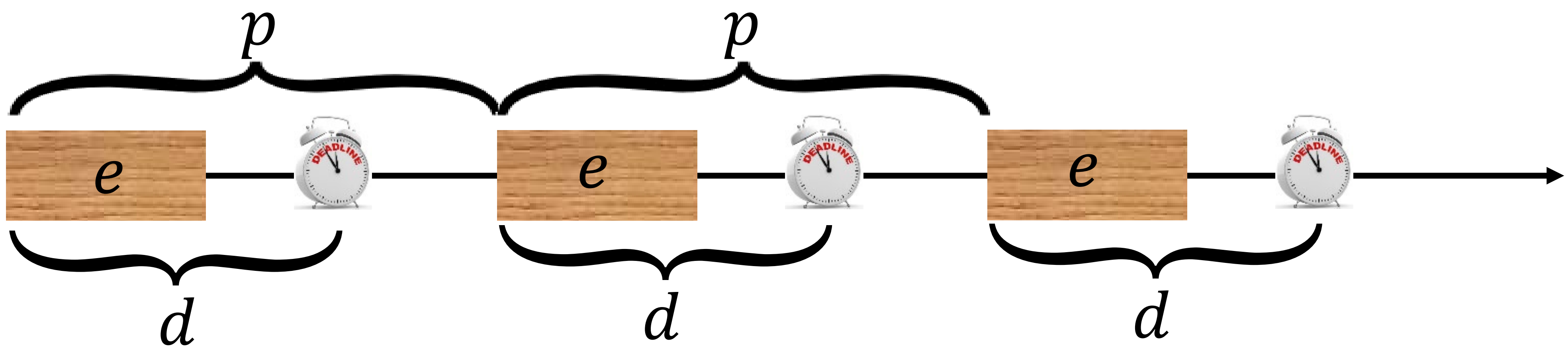
# Outline

- Problem statement
- Main result
- Sketchy proof
- Open problems

# Periodic tasks and sporadic tasks

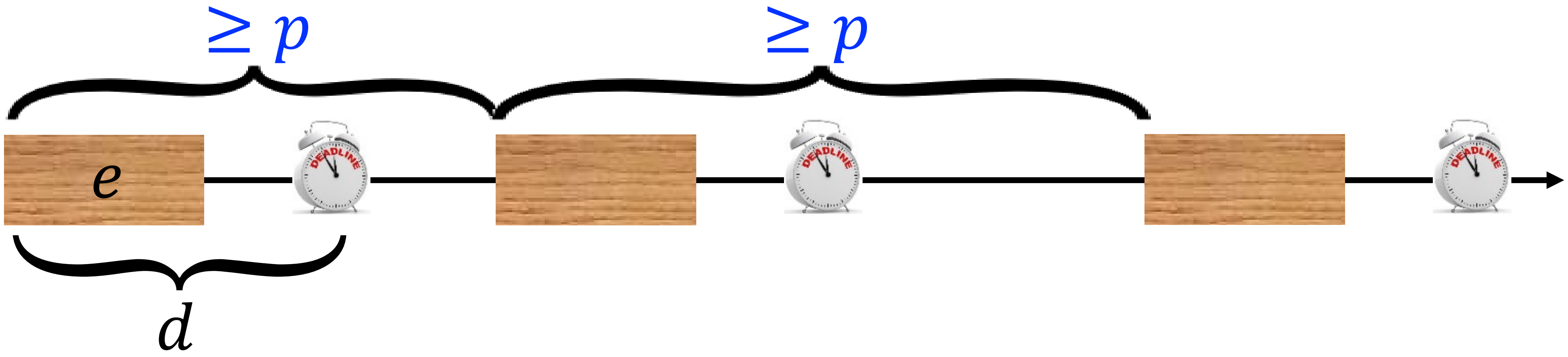
- A task: a sequence of jobs with execution time $e$ and deadline $d$




- Periodic tasks: fixed interarrival time $p$

# Periodic tasks and sporadic tasks

- A task: a sequence of jobs with execution time $e$ and deadline $d$

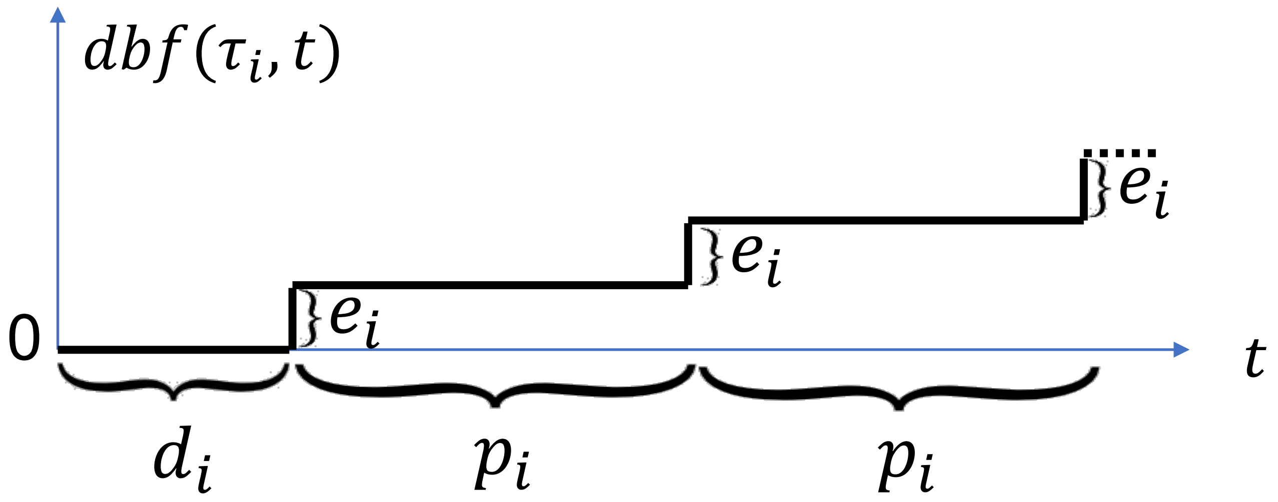



- Periodic tasks: fixed interarrival time $p$
- Sporadic tasks: varying interarrival time $\geq p$

$\tau = (e, d, p)$

# Schedulability testing of $\tau = \{\tau_1, \ldots, \tau_n\}$

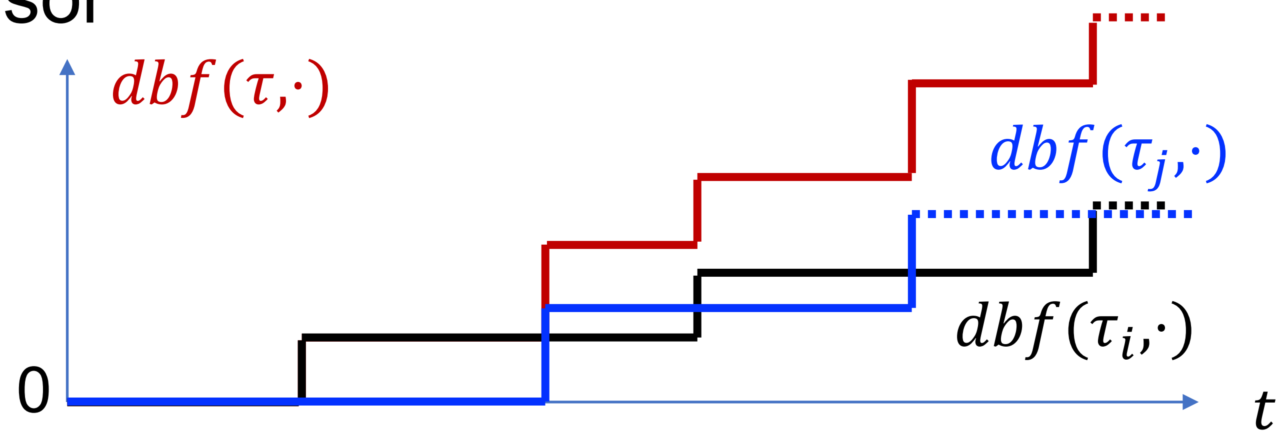
- On a unit-speed uniprocessor
  - Resource demand:



# Schedulability testing of $\tau = \{\tau_1, \dots, \tau_n\}$

- On a unit-speed uniprocessor
  - Resource demand:

$$dbf(\tau, t) = \sum_{\tau_i \in \tau} dbf(\tau_i, t)$$




  - Schedulable $\Leftrightarrow dbf(\tau, t) \leq t$
  - Co-NP-hard to check


F. Eisenbrand and T. Rothvoβ "Edf-schedulability of synchronous periodic task systems is coNP-hard," SODA2010, pp. 1029-1034
P. Ekberg and W. Yi "Uniprocessor Feasibility of Sporadic Tasks Remains coNP-complete Under Bounded Utilization," RTSS2015.

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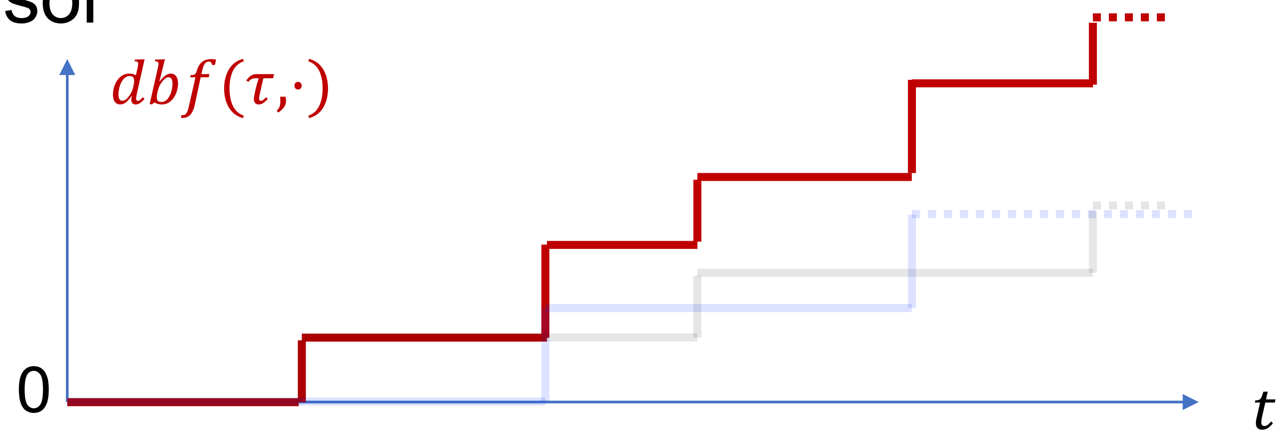
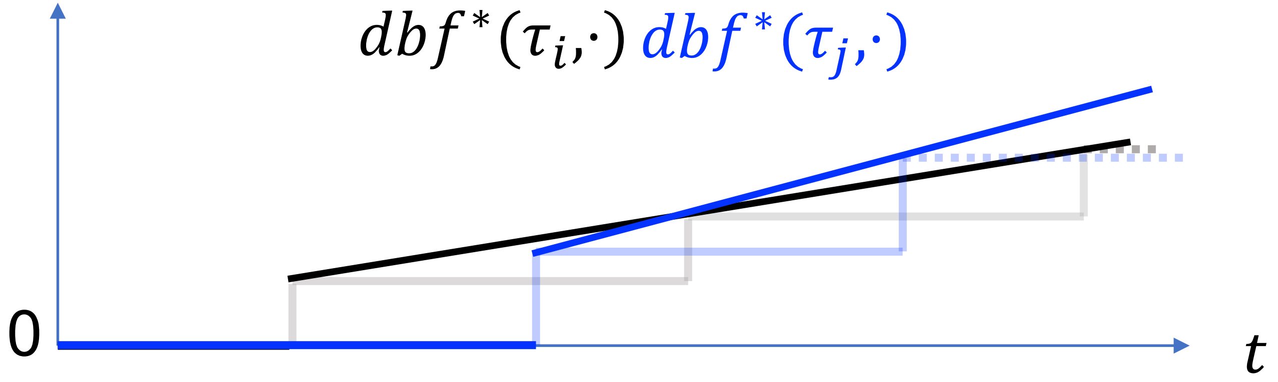



  - Approximation:




K. Albers and F. Slomka, "An Event Stream Driven Approximation for the Analysis of Real-Time Systems," in ECRTS 2004, pp. 187-195.

# Schedulability testing of $\tau = \{\tau_1, \ldots, \tau_n\}$

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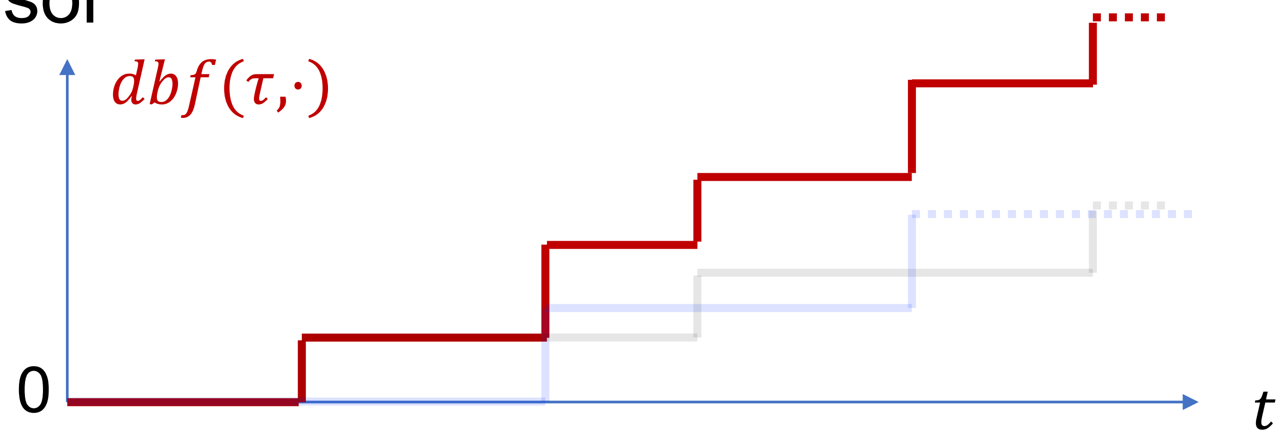
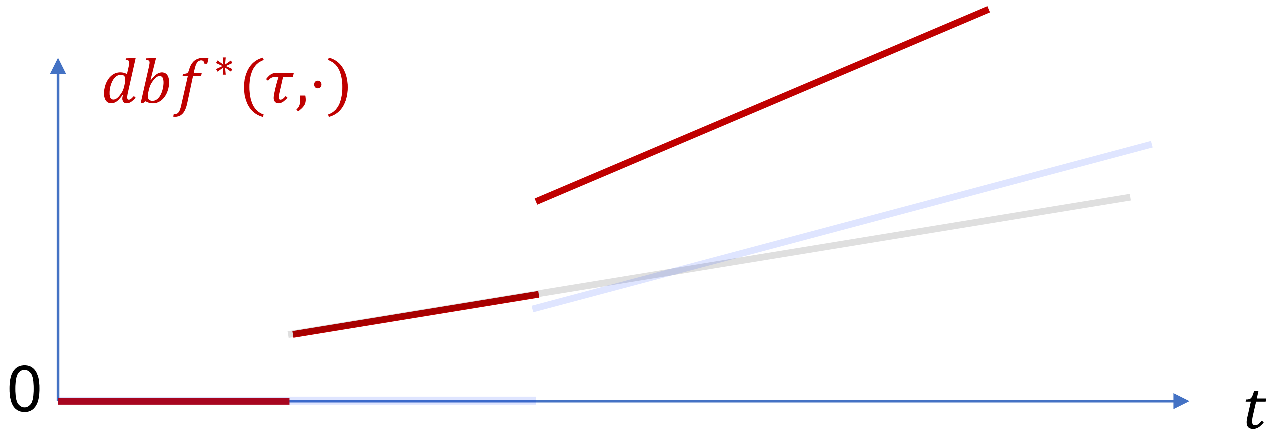
$$dbf(\tau, t) = \sum_{\tau_i \in \tau} dbf(\tau_i, t)$$




  - Approximation:

$$dbf^*(\tau, t) = \sum_{\tau_i \in \tau} dbf^*(\tau_i, t)$$

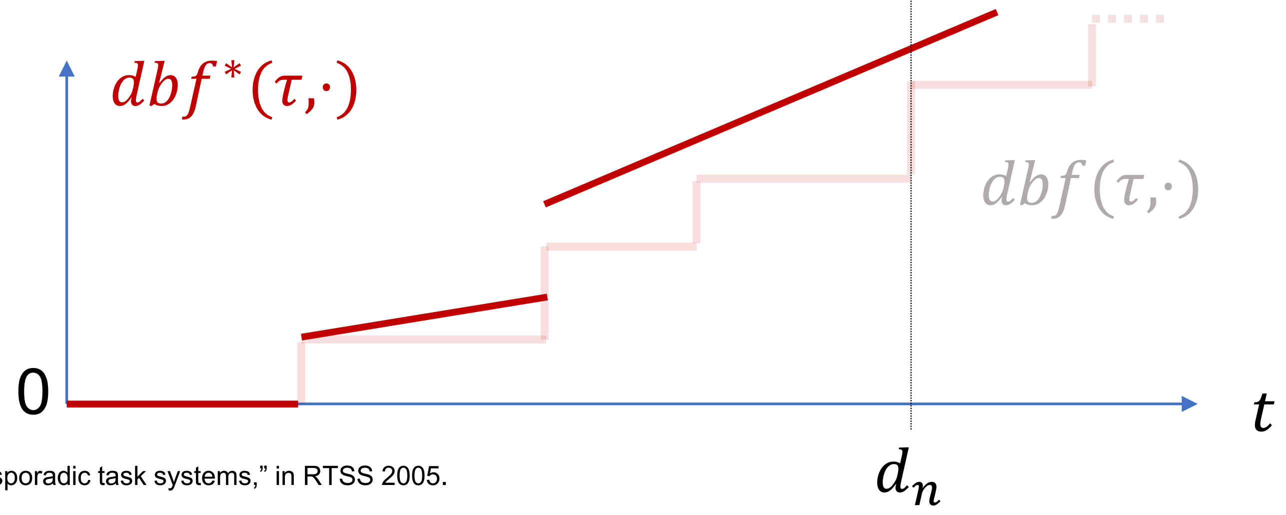



K. Albers and F. Slomka, “An Event Stream Driven Approximation for the Analysis of Real-Time Systems,” in ECRTS 2004, pp. 187-195.

# Schedulability testing of $\tau = \{\tau_1, \dots, \tau_n\}$

- On a speed-1 multiprocessor
  - Partitioned scheduling (Any fit with dbf* works)
  - Speedup bound is $1 + \rho$ [a]

$$\rho = \sup_{\tau : dbf(\tau,t) \le t} \frac{dbf^*(\tau, d)}{d}, d = \max\{d_1, \dots, d_n\}$$




[a] S. Baruah and N. Fisher, "The partitioned multiprocessor scheduling of sporadic task systems," in RTSS 2005.

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$$\rho = \sup_{\tau : dbf(\tau,t) \leq t} \frac{dbf^*(\tau, d)}{d}, d = \max\{d_1, \ldots, d_n\}$$

A schedulability test has **speedup factor**[b] **s**, **s**≥1, if *any* task set that is *schedulable* by *any* algorithm on platform with processors of speed 1, it will be deemed schedulable by this test upon a platform with processors that are ***s*** *times as fast*.

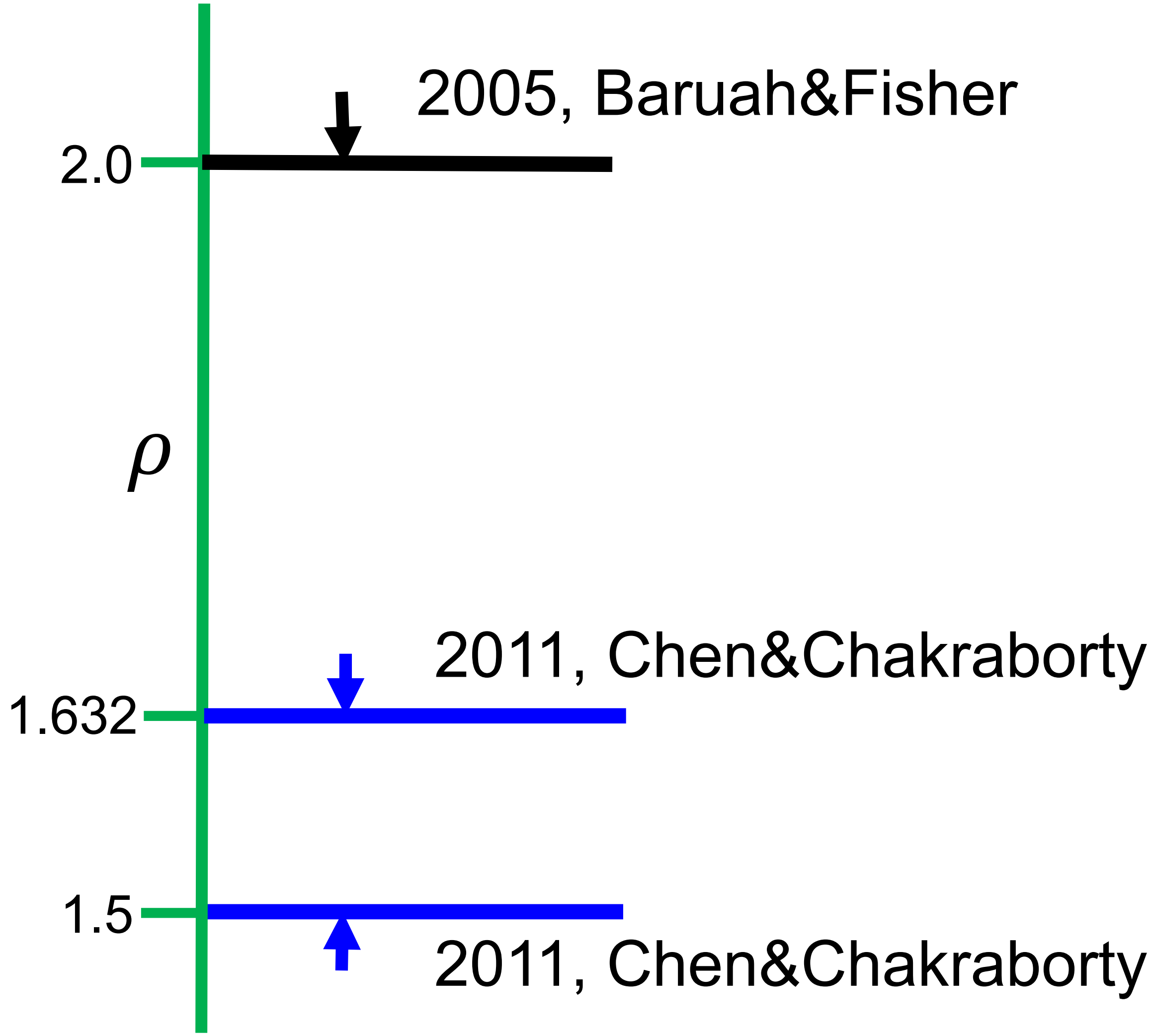
- **Speedup bound** = lower bound of speedup factor
- Major metric & standard tool for evaluating sub-optimality

[a] S. Baruah and N. Fisher, "The partitioned multiprocessor scheduling of sporadic task systems," in RTSS 2005.
[b] B. Kalyanasundaram and K. Pruhs, "Speed is as powerful as clairvoyance," J. ACM, vol. 47, no. 4, pp. 617–643, 2000.

# Ultimate objective

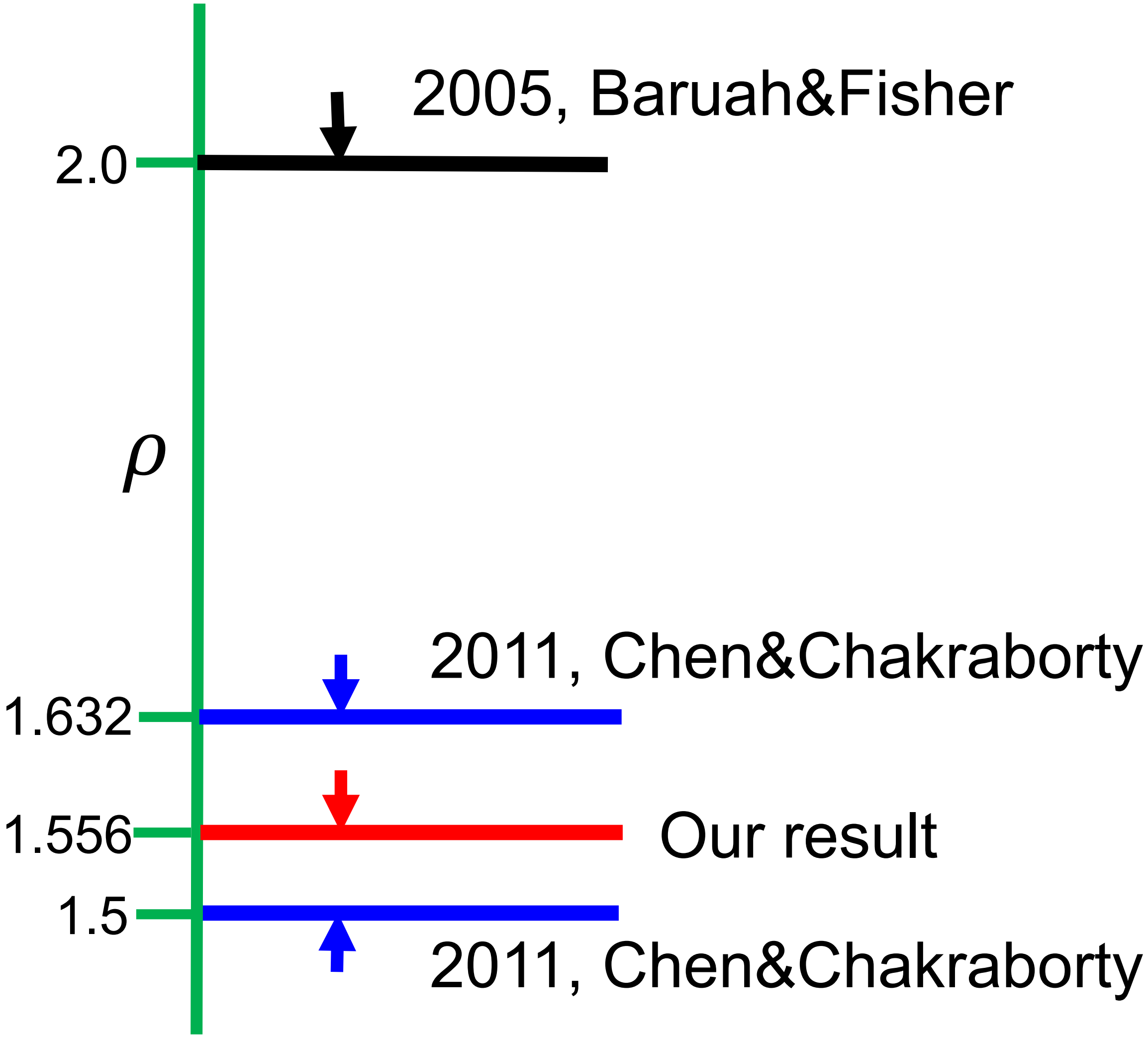
- Figure out $\rho$
- Known results

(Constrained-deadline)



# Main result

- Better upper bound of $\rho$
  - Lower speedup factor of partitioned scheduling



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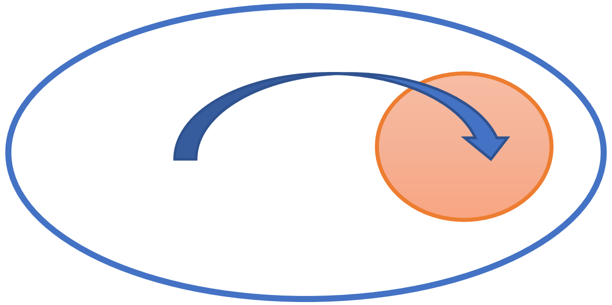
# Main ideas to estimate $\rho$



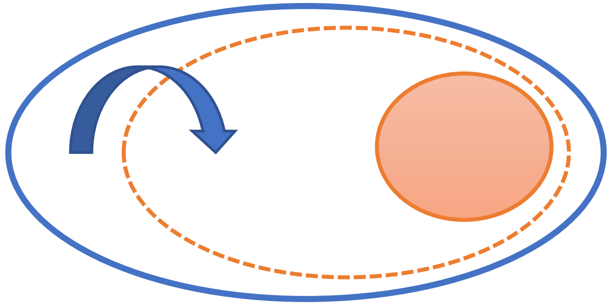
- Principle: Keep $\tau$ feasible while not decreasing Dbf*($\tau$,d_n)
- Details: Fix many parameters
  - Identical execution times: $e_i = 1$
  - Tight deadlines: $d_i = d_{i-1} + e_i = i$
  - Confined periods: $n < p_i + d_i \leq 2n$

} Lossless (identical execution times, tight deadlines)

Lossy (confined periods)

- Reduce the problem to an "easy" math programming

# Step 0: Normalization



- Original objective

$$
\begin{aligned}
\sup \quad & \frac{dbf^*(\tau, d_n)}{d_n}, \\
\textit{subject to} \quad & dbf(\tau, t) \leq t, \quad \forall t > 0 \\
& d_1 \leq d_2 \leq \cdots \leq d_n, \\
& n \in \mathbb{Z}^+, e_i, d_i, p_i \in \mathbb{R}^+, \quad 1 \leq i \leq n.
\end{aligned}
$$

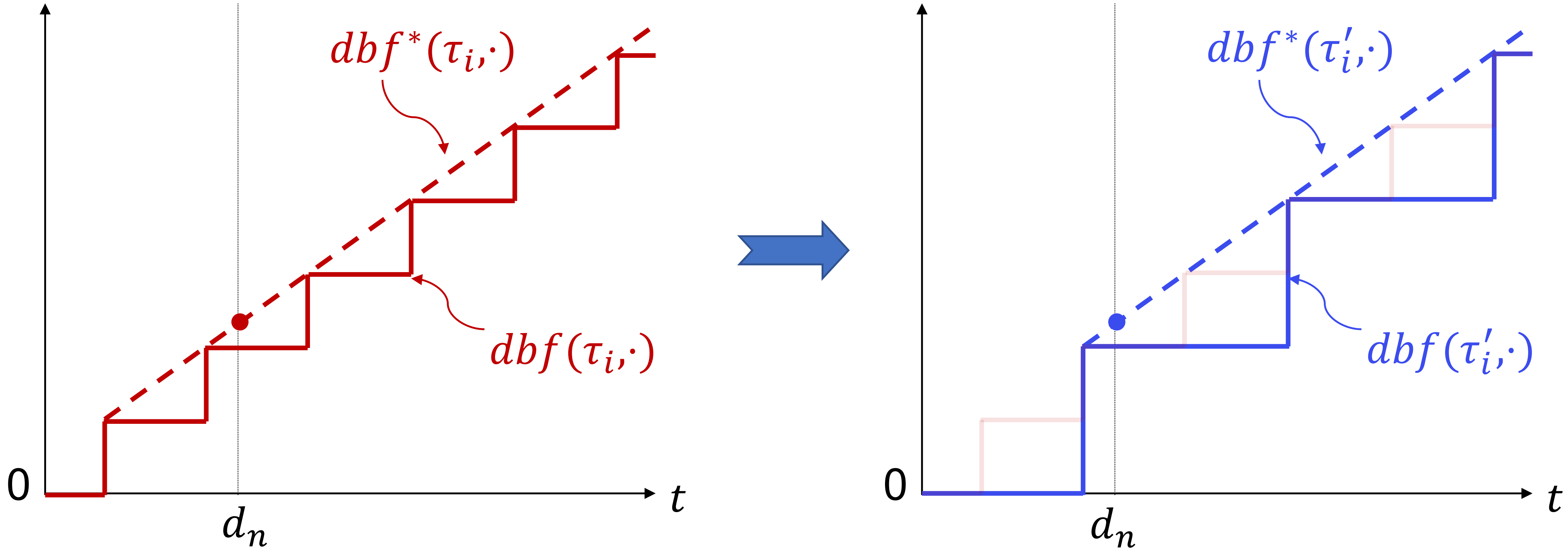
=

- $MP_0$

$$
\begin{aligned}
\sup \quad & \frac{dbf^*(\tau, d_n)}{d_n}, \\
\textit{subject to} \quad & dbf(\tau, t) \leq t, \quad \forall t > 0 \\
& \boxed{d_i + p_i > d_n, \quad 1 \leq i \leq n-1,} \\
& d_1 \leq d_2 \leq \cdots \leq d_n, \\
& n \in \mathbb{Z}^+, e_i, d_i, p_i \in \mathbb{R}^+, \quad 1 \leq i \leq n.
\end{aligned}
$$

J.-J. Chen and S. Chakraborty, “Resource augmentation bounds for approximate demand bound functions,” in RTSS2011.

Let $\tau = \{\tau_1, \ldots, \tau_n\}$ be a feasible solution to the original problem.
Suppose $d_i + p_i \leq d_n$. Transform $\tau_i$ into $\tau_i'$

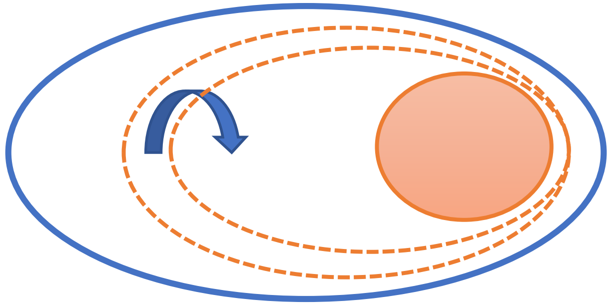



$dbf(\tau_i, t) \geq dbf(\tau_i', t) \Rightarrow \tau' = \{\tau_1', \ldots, \tau_n'\}$ is a feasible solution to $MP_1$

$dbf^*(\tau_i, d_n) = dbf^*(\tau_i', d_n') \Rightarrow$ equal objective values

J.-J. Chen and S. Chakraborty, “Resource augmentation bounds for approximate demand bound functions,” in RTSS2011.

# Step 1: Rationalization



- $MP_0$

$$\begin{aligned} \sup \quad & \frac{dbf^*(\tau, d_n)}{d_n}, \\ \textit{subject to} \quad & dbf(\tau, t) \le t, \quad \forall t > 0 \\ & d_i + p_i > d_n, \quad 1 \le i \le n-1, \\ & d_1 \le d_2 \le \cdots \le d_n, \\ & n \in \mathbb{Z}^+, \boxed{e_i, d_i, p_i \in \mathbb{R}^+,} \quad 1 \le i \le n. \end{aligned}$$

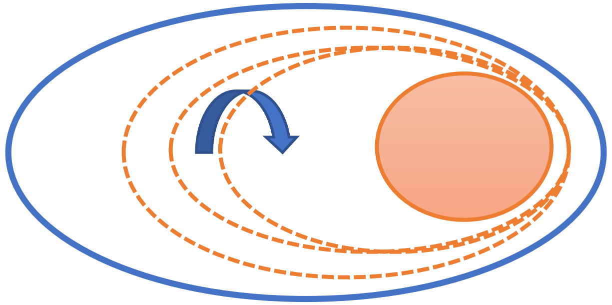
$=$

- $MP_1$

$$\begin{aligned} \sup \quad & \frac{dbf^*(\tau, d_n)}{d_n}, \\ \textit{subject to} \quad & dbf(\tau, t) \le t, \quad \forall t > 0 \\ & d_i + p_i > d_n, \quad 1 \le i \le n-1, \\ & d_1 \le d_2 \le \cdots \le d_n, \\ & n \in \mathbb{Z}^+, \boxed{e_i, d_i, p_i \in \mathbb{Q}^+,} \quad 1 \le i \le n. \end{aligned}$$

Lossless due to continuity

# Step 2: Tight deadlines



- $MP_1$

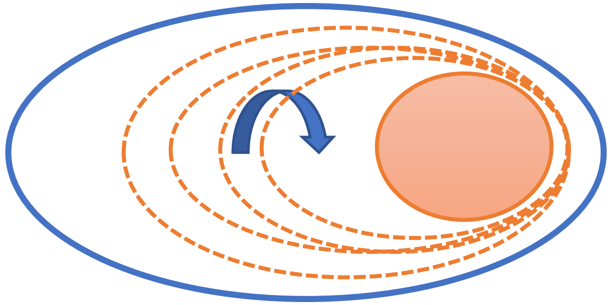
$$
\begin{aligned}
\sup \quad & \frac{dbf^*(\tau, d_n)}{d_n}, \\
\textit{subject to} \quad & dbf(\tau, t) \le t, \quad \forall t > 0 \\
& d_i + p_i > d_n, \quad 1 \le i \le n-1, \\
& d_1 \le d_2 \le \cdots \le d_n, \\
& n \in \mathbb{Z}^+, e_i, d_i, p_i \in \mathbb{Q}^+, \quad 1 \le i \le n.
\end{aligned}
$$

$=$

- $MP_2$

$$
\begin{aligned}
\sup \quad & \frac{dbf^*(\tau, d_n)}{d_n}, \\
\textit{subject to} \quad & dbf(\tau, t) \le t, \quad \forall t > 0 \\
& d_i + p_i > d_n, \quad 1 \le i \le n-1, \\
& d_i = e_i + d_{i-1}, \quad 1 \le i \le n, \\
& n \in \mathbb{Z}^+, e_i, d_i, p_i \in \mathbb{Q}^+, \quad 1 \le i \le n.
\end{aligned}
$$

# Step 3: Identical execution times



- $MP_2$

$$
\begin{aligned}
\sup \quad & \frac{dbf^*(\tau, d_n)}{d_n}, \\
\textit{subject to} \quad & dbf(\tau, t) \le t, \quad \forall t > 0 \\
& d_i + p_i > d_n, \quad 1 \le i \le n-1, \\
& d_i = e_i + d_{i-1}, \quad 1 \le i \le n, \\
& n \in \mathbb{Z}^+, e_i, d_i, p_i \in \mathbb{Q}^+, \quad 1 \le i \le n.
\end{aligned}
$$

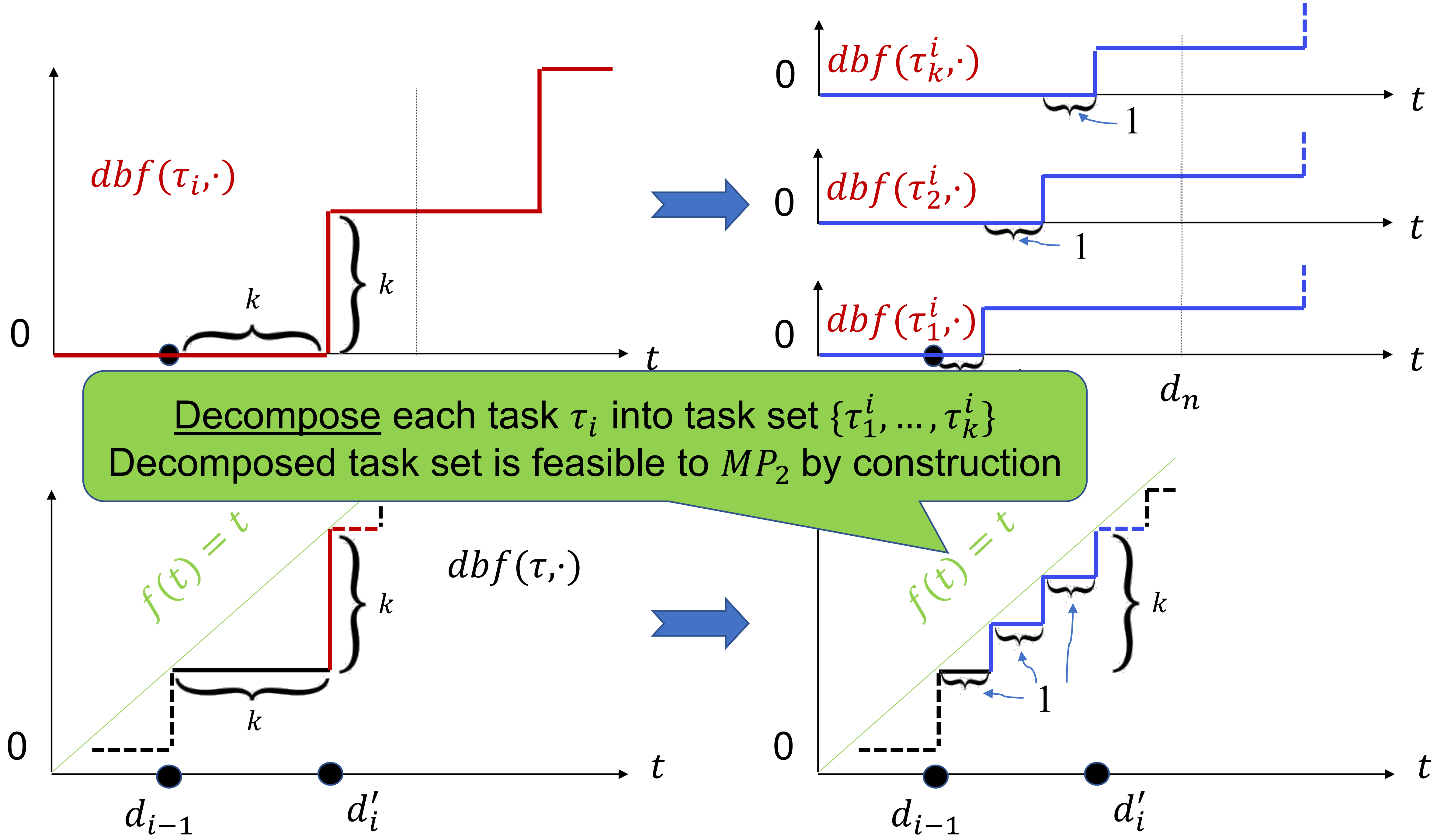
=

- $MP_3$

$$
\begin{aligned}
\sup \quad & \frac{dbf^*(\tau, d_n)}{d_n}, \\
\textit{subject to} \quad & dbf(\tau, t) \le t, \quad \forall t > 0 \\
& d_i + p_i > d_n, \quad 1 \le i \le n-1, \\
& d_i = e_i + d_{i-1}, \quad 1 \le i \le n, \\
& e_i = 1, \quad 1 \le i \le n \\
& n \in \mathbb{Z}^+, e_i, d_i, p_i \in \mathbb{Q}^+, \quad 1 \le i \le n.
\end{aligned}
$$

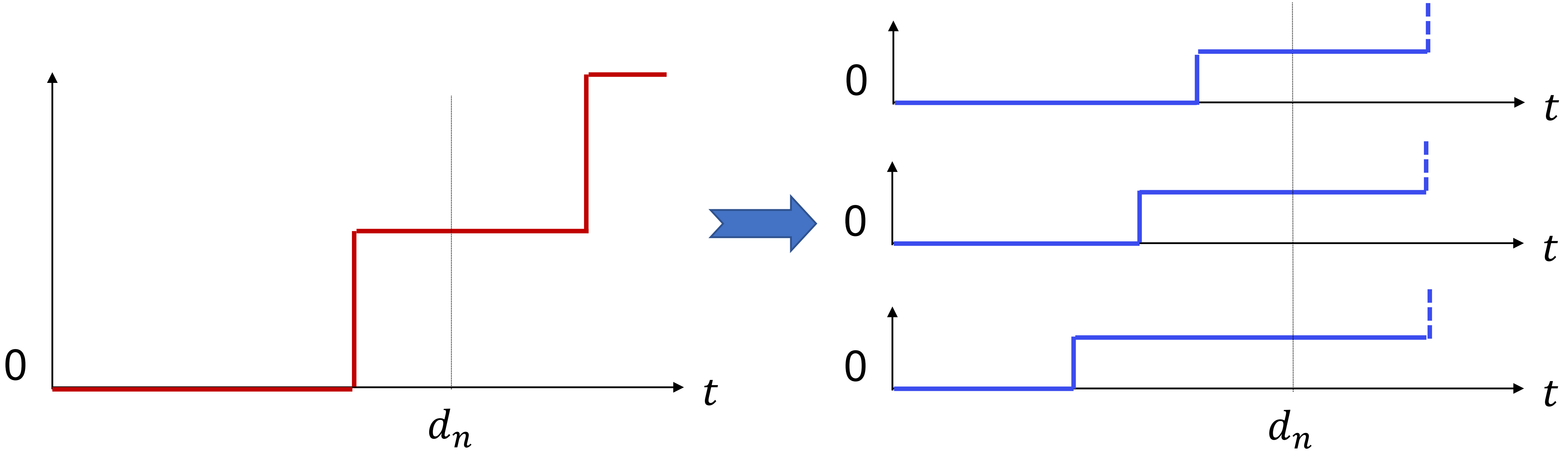
$\mathbf{Z^+}$

- Proportionally scaling all the parameters keeps the feasibility and objective value
- The original parameters are rational



$dbf(\tau_i,\cdot)$
0
$k$
$k$
$t$
$dbf(\tau_k^i,\cdot)$
0
1
$t$
$dbf(\tau_2^i,\cdot)$
0
1
$t$
$dbf(\tau_1^i,\cdot)$
0
$t$
$d_n$
Decompose each task $\tau_i$ into task set $\{\tau_1^i, \ldots, \tau_k^i\}$
Decomposed task set is feasible to $MP_2$ by construction
$f(t) = t$
$dbf(\tau,\cdot)$
$k$
$k$
0
$t$
$d_{i-1}$
$d_i'$
$f(t) = t$
$k$
1
0
$t$
$d_{i-1}$
$d_i'$

$$dbf^*(\tau_i, d_n) \leq \sum_{j=1}^{k} dbf^*(\tau_j^i, d_n)$$



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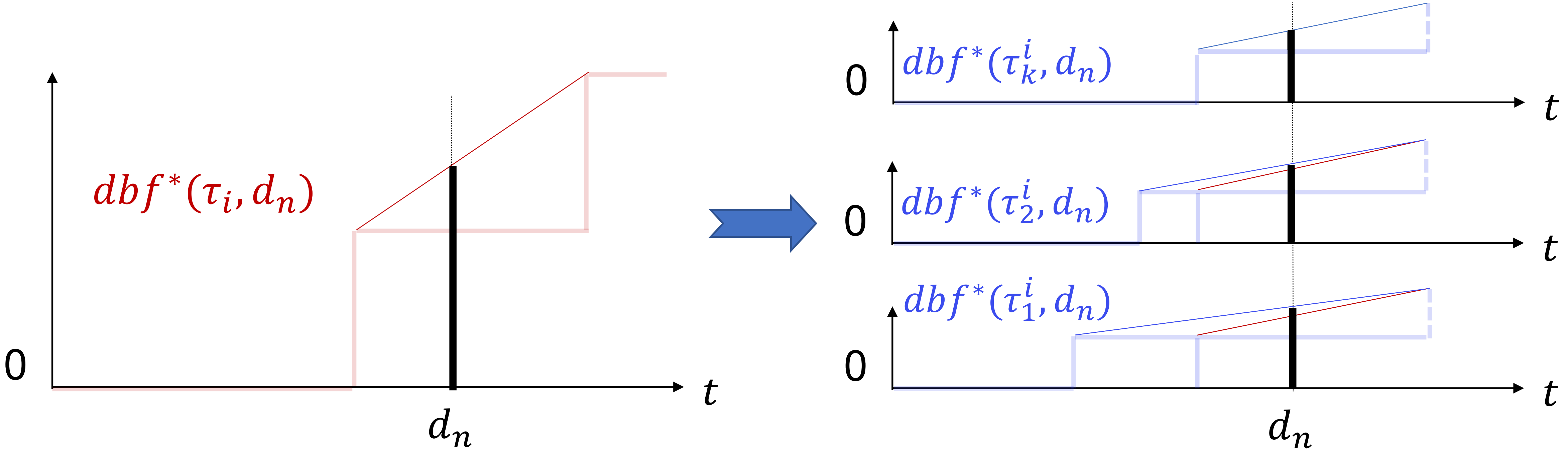
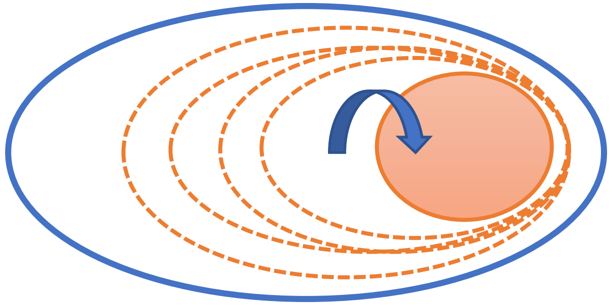


# Step 4: Confined range of periods



- $MP_3$

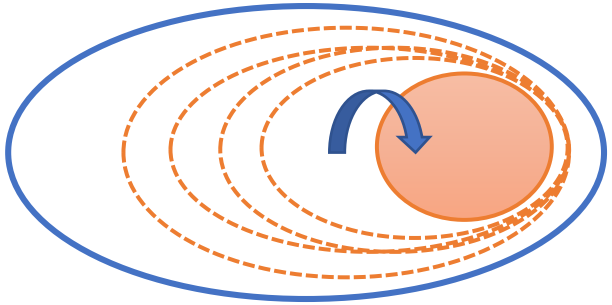
$$\begin{aligned}
\sup \quad & \frac{dbf^*(\tau, d_n)}{d_n}, \\
\textit{subject to} \quad & dbf(\tau, t) \le t, \quad \forall t > 0 \\
& d_i + p_i > d_n, \quad 1 \le i \le n-1, \\
& d_i = e_i + d_{i-1}, \quad 1 \le i \le n, \\
& e_i = 1, \quad 1 \le i \le n \\
& n \in \mathbb{Z}^+, e_i, d_i, p_i \in \mathbb{Q}^+, \quad 1 \le i \le n.
\end{aligned}$$

$$\le$$

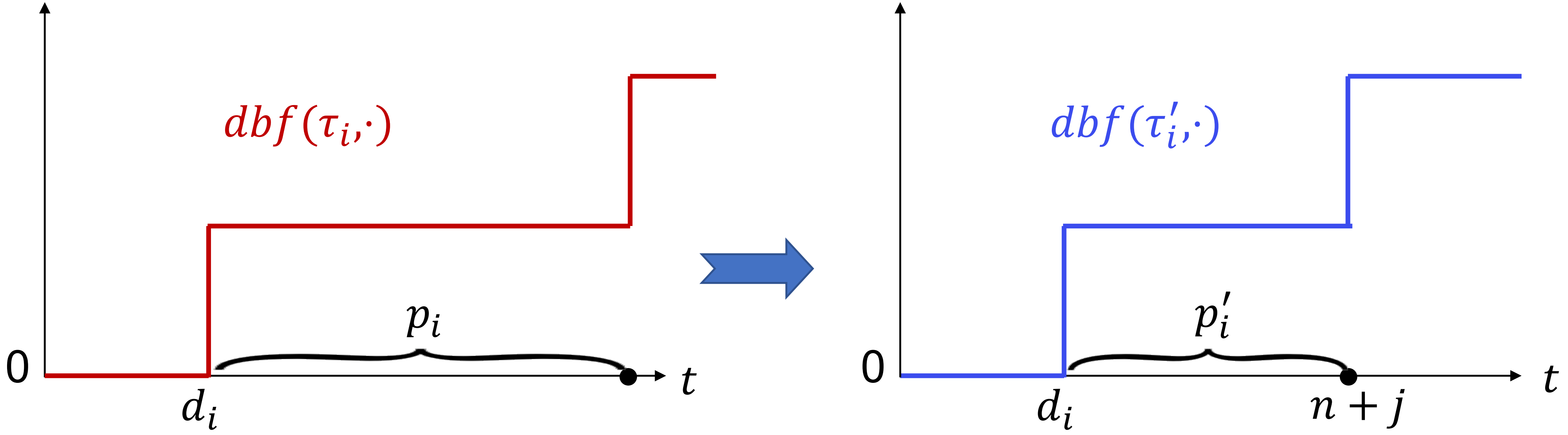
- $MP_4$

$$\begin{aligned}
\sup \quad & \frac{dbf^*(\tau, d_n)}{d_n}, \\
\textit{subject to} \quad & n < d_i + p_i \le 2n \text{ are distinct} \\
& d_i = e_i + d_{i-1}, \quad 1 \le i \le n, \\
& e_i = 1, \quad 1 \le i \le n \\
& n \in \mathbb{Z}^+, e_i, d_i, p_i \in \mathbb{Q}^+, \quad 1 \le i \le n.
\end{aligned}$$

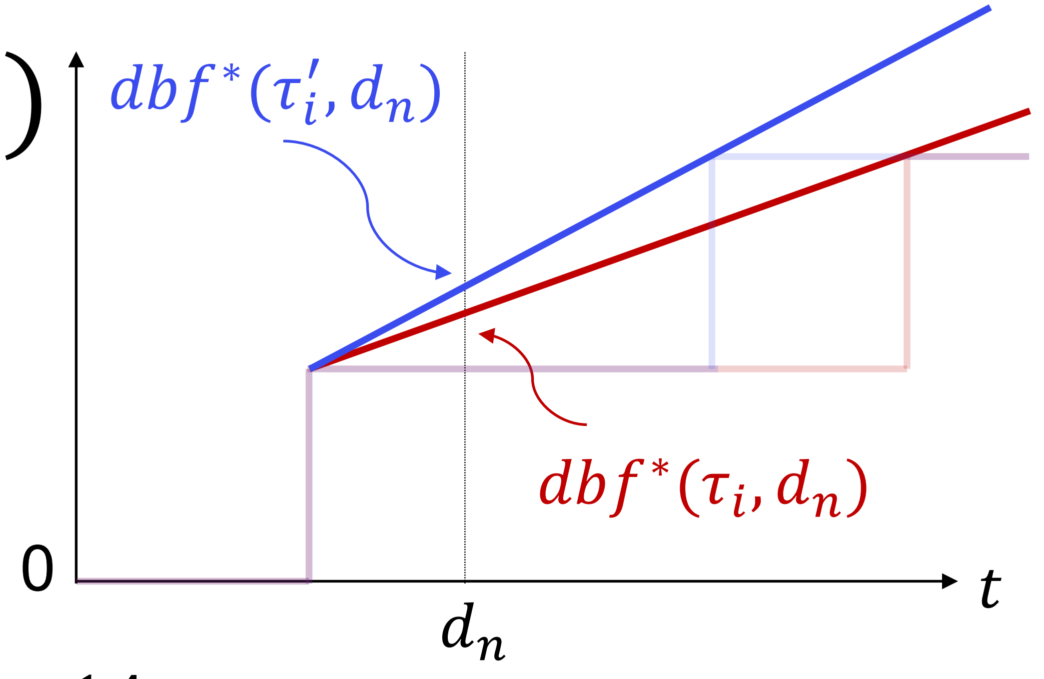
# Basic idea



- Let $\tau = \{\tau_1, \ldots, \tau_n\}$ be a feasible solution to $MP_3$
- For any task $\tau_i = (e_i, d_i, p_i)$, suppose $d_i + p_i$ is the $j$th smallest.
  - $d_i + p_i \geq n + j$ since $d_i + p_i \geq dbf(\tau, d_i + p_i) \geq n + j$
- Transform $\tau_i$ to $\tau_i' = (e_i, d_i, p_i')$ such that $d_i + p_i' = n + j$
- $\tau' = \{\tau_1', \ldots, \tau_n'\}$ is a feasible solution to $MP_4$



$$dbf^*(\tau_i, d_n) \le dbf^*(\tau'_i, d_n)$$




- $\rho$ is upper bounded by the optimum value $\rho'$ of $MP_4$

- $\rho' \le \sup_{p_1+\cdots+p_n=n^2} 2 - \frac{\sum_{i=1}^{n} \frac{i}{p_i}}{n}$
- $\sum_{i=1}^{n} x_i = n^2 \Longrightarrow \sum_{i=1}^{n} \frac{i}{x_i} \ge \frac{4n}{9}$

$$\Longrightarrow \rho' \le \frac{14}{9}$$

$$\Longrightarrow \rho \le \frac{14}{9}$$

***MP*$_4$**

$$\sup \quad \frac{dbf^*(\tau, d_n)}{d_n},$$

*subject to* $n < d_i + p_i \le 2n$ are distinct

$d_i = e_i + d_{i-1}, \quad 1 \le i \le n,$

$e_i = 1, \quad 1 \le i \le n$

$n \in \mathbb{Z}^+, e_i, d_i, p_i \in \mathbb{Q}^+, \quad 1 \le i \le n.$

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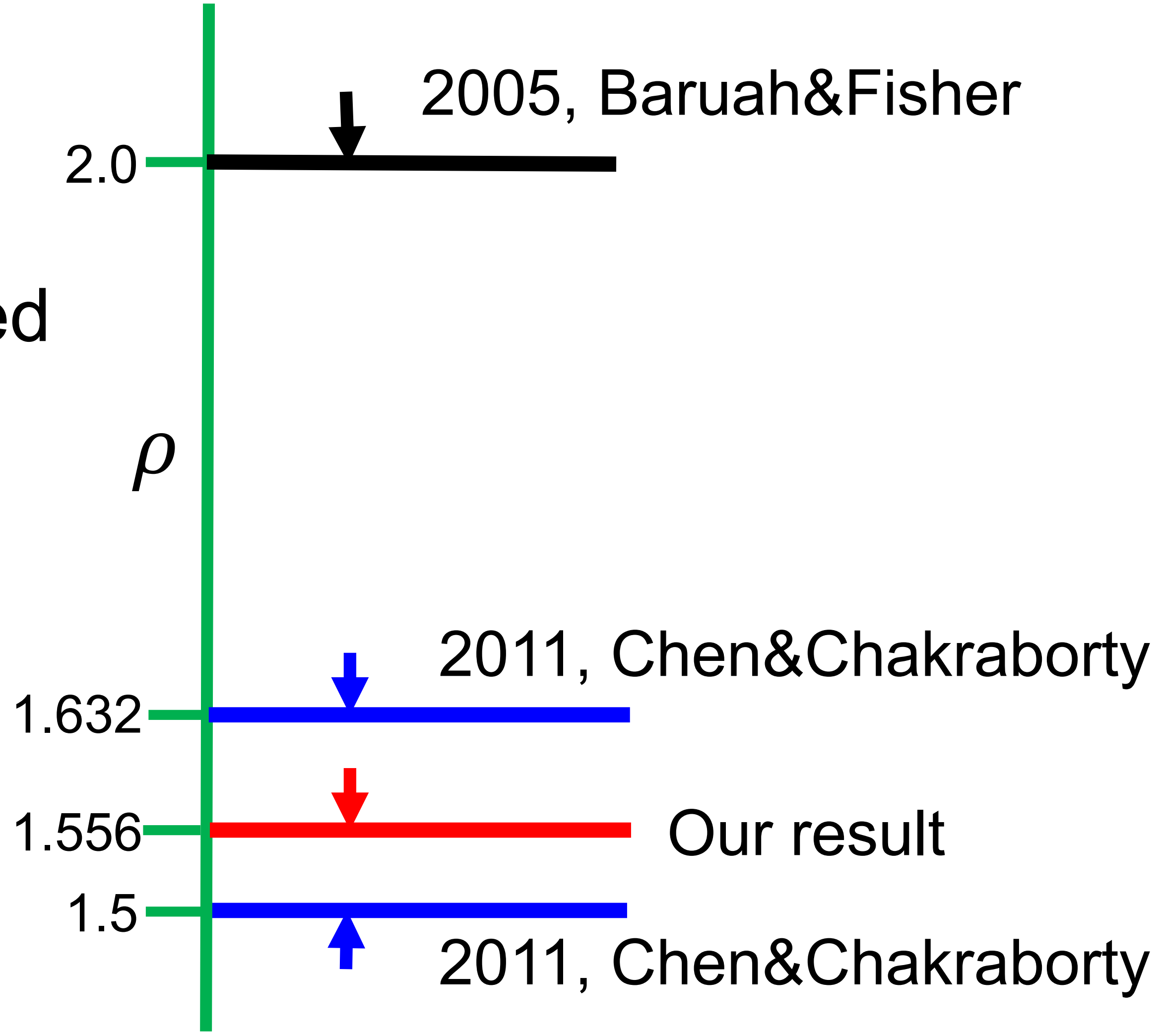
- Conclusion

# Conclusion

- Demand/resource $\rho$
- Speedup factor of partitioned EDF: 2.5556.
- WiP: Arbitrary deadline (uniproc only)



# Thank You!

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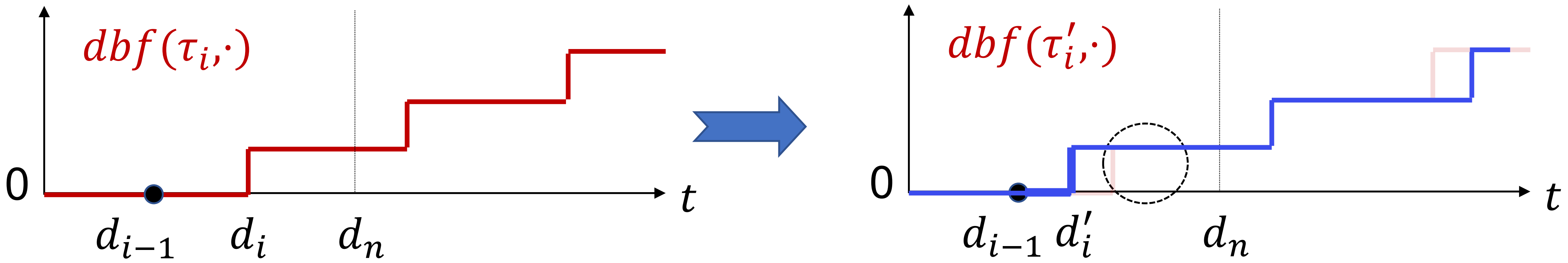
# Speedup / Resource Augmentation Bound

A schedulability test has **speedup factor**[a] **s**, **s**≥1, if *any* task set that is *schedulable* by *any* algorithm on platform with processors of speed 1, it will be deemed schedulable by this test upon a platform with processors that are **s** *times as fast*.

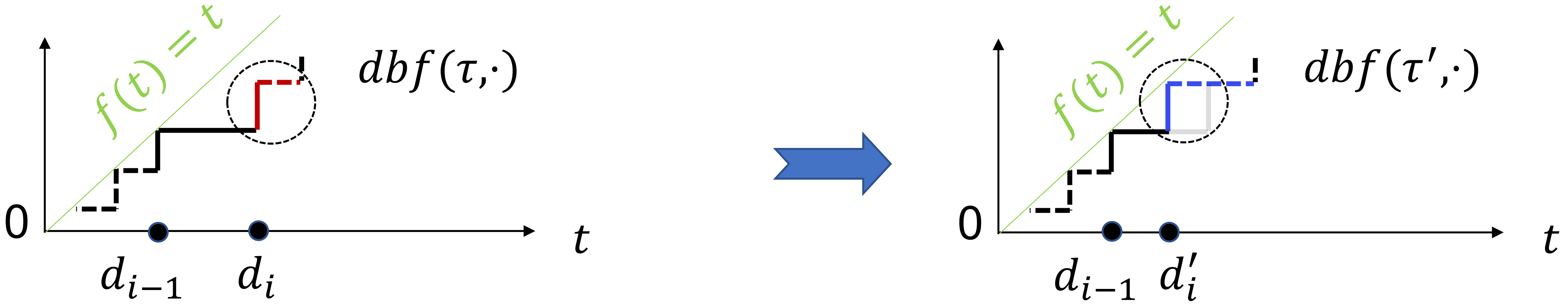
- Speedup bound means a lower bound of speedup factor
- Major metric & standard tool for evaluating sub-optimality
- Potential pitfalls

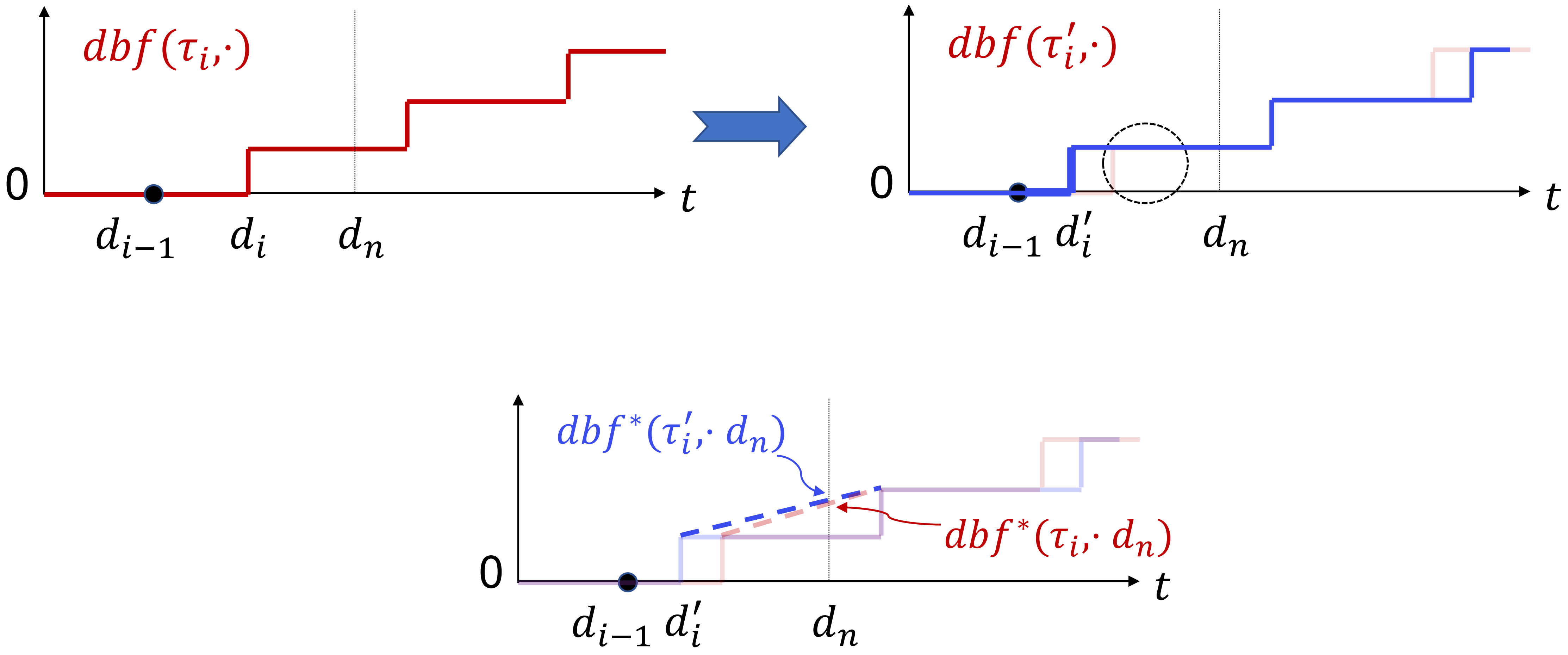
Z. Guo, "Regarding the optimality of speedup bounds of mixed-criticality schedulability tests," Dagstuhl Seminar 17131, 2017.
J.-J. Chen et al., "On the Pitfalls of Resource Augmentation Factors and Utilization Bounds in Real-Time Scheduling," in ECRTS 2017, pp. 9:1–9:25.
K. Agrawal and S. Baruah, "Intractability issues in mixed-criticality scheduling," in ECRTS'18.

Let $\tau = \{\tau_1, \dots, \tau_n\}$ be a feasible solution to $MP_1$.
Chose the smallest $i$ s.t. $d_i \neq e_i + d_{i-1}$. Transform $\tau_i$ into $\tau_i'$




$dbf(\tau', t) \leq t$ holds by construction






$$dbf^*(\tau_i, d_n) \le dbf^*(\tau_i', d_n)$$