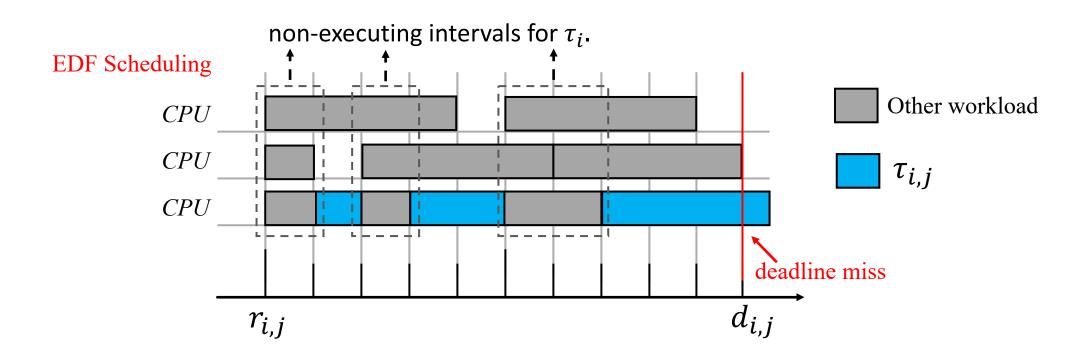
New Analysis Techniques for Supporting Hard Real-Time Sporadic DAG Task Systems on Multiprocessors

Zheng Dong and Cong Liu

According to [1], it is generally impossible to prove utilization bounds for parallel tasks.

[1] S. K. Dhall and C. Liu, "On a real-time scheduling problem," Operations Research, vol. 26, no. 1, pp. 127–140, 1978.

A high-level review of Theodore P. Baker's analysis for ordinary sporadic tasks

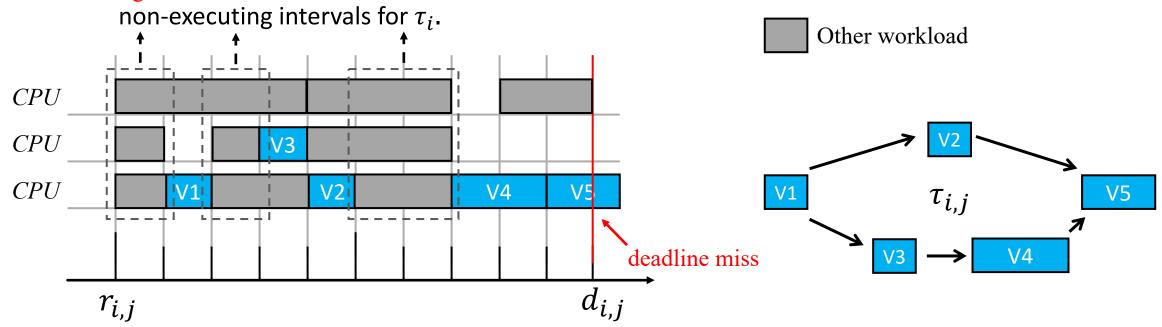


$$W > m \times (d_i - e_i) + e_i$$

$$\sum_{k=1}^{n} \beta_k \le m - (m-1) \times u_i$$

If we directly apply Theodore P. Baker's analysis to sporadic DAG tasks

EDF Scheduling

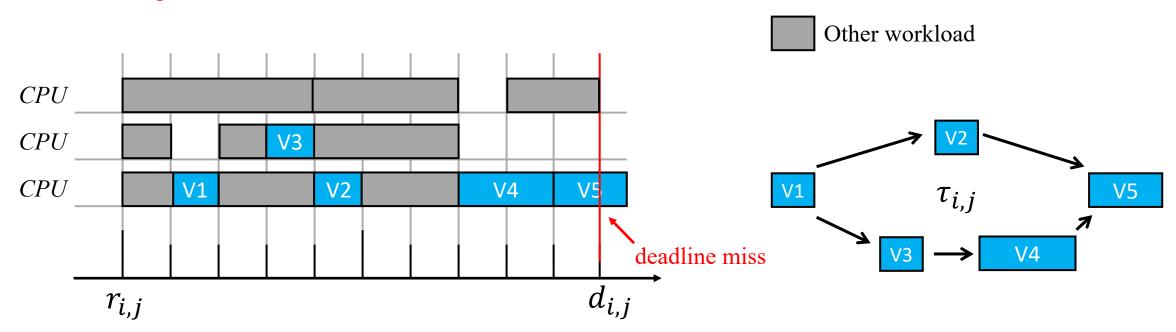


$$W > m \times (d_i - e_i) + e_i \qquad \qquad \square > \qquad \sum_{k=1} \beta_k \le m - (m-1) \times u_i$$

$$u_i > 1$$

Subtask Ordering: The Lazy-Cpath Policy

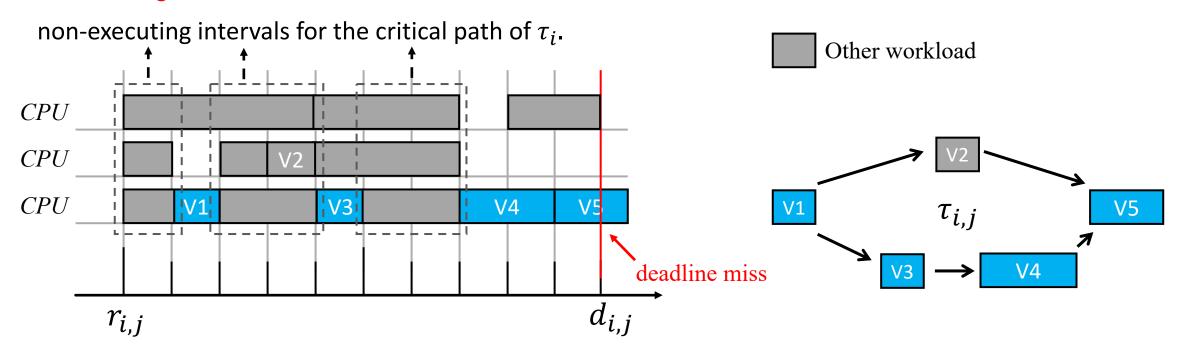
EDF Scheduling



Under the Lazy-Cpath policy, at any time instance, ready jobs of subtasks on a DAG's critical path has the lowest priority among all ready jobs of subtasks belonging to the same DAG.

Subtask Ordering: The Lazy-Cpath Policy

EDF Scheduling



$$W > m \times \left(d_i - e_i^{Cpath}\right) + e_i^{Cpath} \quad \Longrightarrow \quad \sum_{k=1}^n \eta_k \le m - (m-1) \times \frac{e_i^{Cpath}}{p_i}$$